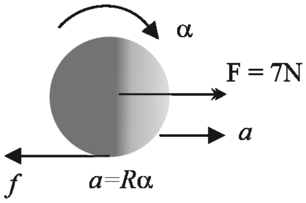
Solutions to JEE Main Home Practice Test - 3 | JEE - 2024

PHYSICS

SECTION-1

1.(B) $f_{\max} = \mu mg = 0.1 \times 10 \times 10 = 10N$

Requirement of friction



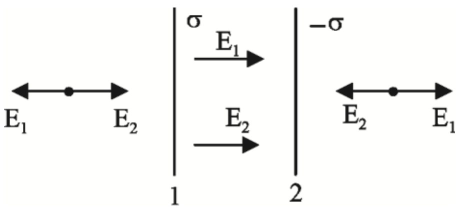
$$\left(\frac{7-f}{10}\right) = \frac{R.(f.R)}{2/5mR^2} = \frac{2.5f}{m} = \frac{2.5f}{10}$$

Solving this equation, we get

$f = 2$

Since, $f < f_{\max}$, therefore 2 N frictional force will act.

2.(B)



$\because\ E_1 = E_2 = \dfrac{\sigma}{2\in_0}$, By superposition, we get option 2.

3.(A) Let us conserve angular momentum of $+2q$ about the point at $+Q$.

$$mv_1r_1\sin\theta_1 = mv_2r_2\sin\theta_2$$

$$mvR\sin150° = m\left(\frac{v}{\sqrt{3}}\right)r_{\min}\sin 90°$$

$$r_{\min} = \frac{\sqrt{3}}{2}R$$

4.(A) $a = bt \qquad \therefore\ \dfrac{dv}{dt} = bt$

Or $\displaystyle\int_{v_0}^{v} dv = \int_0^t (bt)\,dt \qquad \therefore\ v = v_0 + \frac{bt^2}{2}$

$$s = \int_0^t v\,dt = \int_0^t \left(v_0 + \frac{bt^2}{2}\right)dt = v_0t + \frac{bt^3}{6}$$

5.(A) $v = \dfrac{c}{\sqrt{\mu_r \in_r}} = \dfrac{3\times10^8}{\sqrt{64}} = \dfrac{3}{8}\times10^8 = 0.375\times10^8 = 3.75\times10^7\,m/s$

6.(A) Pitch $= \dfrac{5\,mm}{10} = 0.5\,mm$.

Least count $= \dfrac{0.5\,mm}{50} = \dfrac{5}{500}mm = \dfrac{1}{100}mm = \dfrac{1}{1000}cm = 0.001cm$

Assertion is correct, Reason is also correct.

7.(C) Heat energy $= \dfrac{1}{4} \times$ Kinetic energy

$$mc\Delta T = \frac{1}{4} \times \frac{1}{2} mv^2$$

$$0.1 \times 420 \times \Delta T = \frac{1}{8} \times 1.5 \times (60)^2$$

$$\Delta T = 16.07°C$$

8.(D)

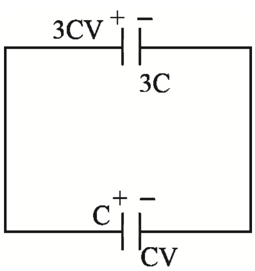
$$C_1 = \frac{K \in_0 A}{d}\; C_2 = \frac{2K \in_0 A}{2d}\; C_3 = \frac{5K \in_0 A}{3d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{A \in_0 K} + \frac{2d}{2A \in_0 K} + \frac{3d}{5A \in_0 K}$$

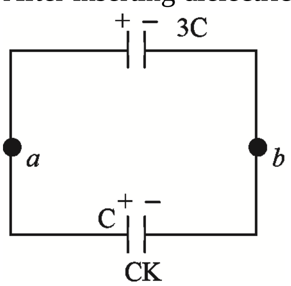
$$= \frac{d}{A \in_0 K}\left(1 + \frac{2}{2} + \frac{3}{5}\right) = \frac{d}{A \in_0 K} \times \frac{13}{5}$$

Thus $C_{eq} = \dfrac{A \in_0 K}{d} \times \dfrac{5}{13}$

9.(D) Just after removing cell



After inserting dielectric



Conservation of charge

$$3CV + CV = 3CV_{ab} + CkV_{ab}$$

$$V_{ab} = \frac{4V}{3+K}$$

10.(D) Angle of projection from B is 45°. As the body is able to cross the well of diameter 40m, hence

$R = \dfrac{v^2}{g}$ R or $v = \sqrt{gR}$ or $v = \sqrt{10 \times 40} = 20\,ms^{-1}$

On the inclined plane, the retardation is

$$g \sin\alpha = g \sin 45° = \frac{10}{\sqrt{2}} ms^{-2}$$

Using, $v^2 - u^2 = 2ax$

$$(20)^2 - u^2 = 2 \times \left(-\frac{10}{\sqrt{2}}\right) \times 20\sqrt{2}$$

$$\therefore \quad u = 20\sqrt{2} ms^{-1}, i.e., V = 20\sqrt{2}\, ms^{-1}$$

11.(B) $T \propto U$

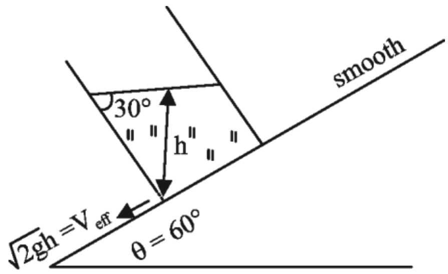
$$T_C = T_D = T_0 = 300K$$

$$T_A = T_B = 2T_0 = 600K$$

$$Q_{net} = Q_{AB} + Q_{BC} + Q_{CD} + Q_{DA} = nRT_A \ln\left(\frac{V_f}{V_i}\right) + nC_V\left(T_C - T_B\right) + nRT_C \ln\left(\frac{V_f}{V_i}\right) + nC_V\left(T_A - T_D\right)$$

$$= (1)(R)(600)\ln(2) + (1)(C_V)(-300) + (1)(R)(300)\ln\left(\frac{1}{2}\right) + (1)(C_V)300 = 300R\ln 2$$

12.(C) Thrust force = $\rho \times \left(area\, of\ cross - section\right)v^2 = \rho a 2gh$



For not slipping down,

$$\rho a 2gh = Mg\sin 60°$$

$$h = \frac{\sqrt{3}}{2}\frac{Mg}{\rho a.2g} = \frac{\sqrt{3}}{4}\frac{M}{\rho a}$$

13.(B) Let velocity of m just after collision is v. Then, from conservation of momentum we have,

$$m_1 v_1 = mv + m_1\frac{v_1}{3} \Rightarrow \therefore\ v = \frac{2}{3}\frac{m_1 v_1}{m}$$

Now, for just completing the circle,

$$v = \sqrt{5gl}$$

$$\frac{2}{3}\frac{m_1 v_1}{m} = \sqrt{5gl}$$

$$v_1 = \frac{3m}{2m_1}\sqrt{5gl}$$

14.(C) $X_L = \omega L = \left(5\times10^{-3}\right)2000 = 10\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000\times\left(50\times10^{-6}\right)} = 10\Omega$$

Since, $X_L = X_C$ circuit is in resonance.

$$Z = R = (6+4) = 10\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{10\sqrt{2}}{10} = 1.414A$$

This is also the reading of ammeter.

$$V = 4I_{rms} = 5.6\,volt$$

15.(A) $\lambda_e = \dfrac{h}{\sqrt{2mK}}$

Also for photon $K = \dfrac{hc}{\lambda_p}$

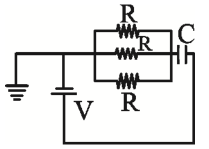
$$\lambda_e = \frac{h\sqrt{\lambda_p}}{\sqrt{2mhc}}$$

$\lambda_p \propto \lambda_e^2$

16.(B) PE $= 81\%$ of total energy.

$\frac{1}{2}m\omega^2 x^2 = 0.81 \times \frac{1}{2}m\omega^2 a^2 \Rightarrow x = 0.9a$

17.(C) The simple circuit is as shown below.



$R_{net} = \frac{R}{3}$

$\tau_C = CR_{net} = \frac{CR}{3}$

$q = q_0\left(1 - e^{-t/\tau C}\right)$

Where, $q_0 = CV$

18. (A) $dQ = -dU$ or $dU + dW = -dU$

Or $2dU + dW = 0$

$\therefore\ 2\left[nC_V dT\right] + pdV = 0$

Or $2\left[n\left(\frac{R}{\gamma - 1}\right)dT\right] + pdV = 0$

Or $\frac{2nRdT}{\gamma - 1} + \left(\frac{nRT}{V}\right)dV = 0$

Or $\left(\frac{2}{\gamma - 1}\right)\frac{dT}{T} + \frac{dV}{V} = 0$

Integrating we get,

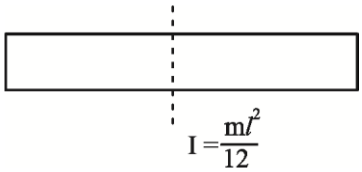
$\left(\frac{2}{\gamma - 1}\right)\ln(T) + \ln(V) = \ln(C)$

Solving we get

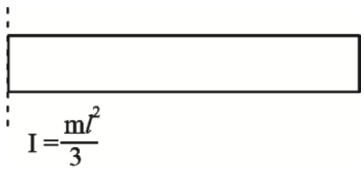
$TV^{\frac{\gamma - 1}{2}} = \text{constant}$

19. (D) For a rod of mass $= m$, length $= l$; formula for moment of inertia are

(A)



(B)



20. (C) $2I_0 = 4I_0 \cos^2\left(\frac{\phi_1}{2}\right)$

$$\phi_1 = \left(\frac{\pi}{2}\right) = \left(\frac{2\pi}{\lambda}\right)(\Delta x_1) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{y_1 d}{D}\right)$$

$$\therefore\ y_1 = \left(\frac{\lambda D}{4d}\right) = \frac{\beta}{4}$$

Further, $I_0 = 4I_0 \cos^2\left(\frac{\phi_2}{2}\right)$

$$\phi_2 = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right)(\Delta x_2) = \left(\frac{2\pi}{\lambda}\right)\left(\frac{y_2 d}{D}\right)$$

$$y_2 = \left(\frac{\lambda D}{3d}\right) = \frac{\beta}{3}$$

$$\Delta y = y_2 - y_1 = \frac{\beta}{12}$$

SECTION - 2

1.(66) $\lambda_{particle} = \frac{h}{p} = \frac{h}{6.6\times10^{-27}\times10^{6}} = \frac{h}{6.6\times10^{-21}}$

$$\lambda_{photon} = \frac{h}{p} = \frac{h}{10^{-22}}$$

$$\frac{\lambda_{photon}}{\lambda_{particle}} = \frac{6.6\times10^{-21}}{10^{-22}} = 66$$

2.(4) Longest wavelength of Lyman series means,
Minimum energy corresponding $n = 2 \, to \, n = 1$.

$$(E_2 - E_1)_H = (E_n - E_2)_{He^+}$$

$$\frac{-13.6}{2^2} + \frac{13.6}{1^2} = -\frac{13.6(Z)^2}{n^2} + \frac{13.6(Z)^2}{2^2}$$

Putting $Z = 2$, we get $n = 4$.

3. (15) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\left(\frac{YA}{l}\right)x^2 = mv^2$$

$$\frac{0.5\times10^9\times10^{-6}}{0.1}\times0.03\times0.03 = 20\times10^{-3}v^2$$

$$v = 15\,m/s$$

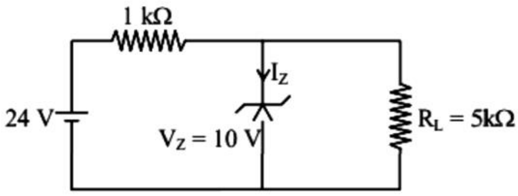
4.(0) In second position, $\Delta\phi = 0$

$$\therefore\ |Q_2| = \frac{\Delta\phi}{R} = 0$$

5.(15) $\frac{1}{f} = \left(\frac{1}{1.5} - 1\right)\left(\frac{1}{10} - \frac{1}{-10}\right)$

Solving, we get $f = -15\,cm$

6.(120)



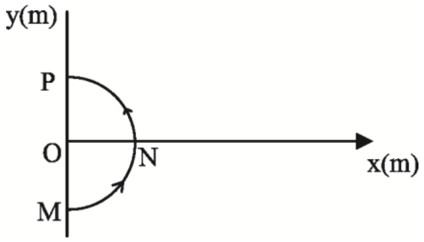
$$i = \frac{10V}{5k\Omega} = 2mA$$

$$I = \frac{14V}{1k\Omega} = 14mA$$

$$\therefore \quad I_z = 12mA$$

$$\therefore \quad P = I_z V_z = 120\,mW$$

7.(60) **At** $x = 0$,



$$y = \pm 2\,m$$

$$F_{MNP} = F_{MP} = i\left[MP \times B\right] = 3\left[\left(4j\right)\times\left(5k\right)\right] = 60\hat{i}$$

8.(625) $F = -\frac{dU}{dr} = -6r^2$

Now $\frac{mv^2}{r} = 6r^2$

$\therefore \frac{1}{2}mv^2 = 3r^3$

$E = K + U = 3r^3 + 2r^3 = 5r^3$

At $r = 5\,m, E = 625\,J$

9.(7750) $\because \lambda = \frac{12400}{1.6}\,\text{Å} = 7750\text{Å}$

10.(20) Length of open organ pipe, $l = 80cm = 0.8m$

So, first overtone $(n = 2)$,

$$f_1 = \frac{nv}{2l} = \frac{2v}{2\times 0.8} = \frac{5v}{4}$$

Length of closed organ pipe, $l = ?$

So, fundamental frequency $(n = 1)$, $f_2 = \frac{nv}{4l} = \frac{v}{4l}$

Given, $f_1 = f_2$

Hence, $\frac{5v}{4} = \frac{v}{4l}$

$\Rightarrow \quad l = 0.2m = 20cm$

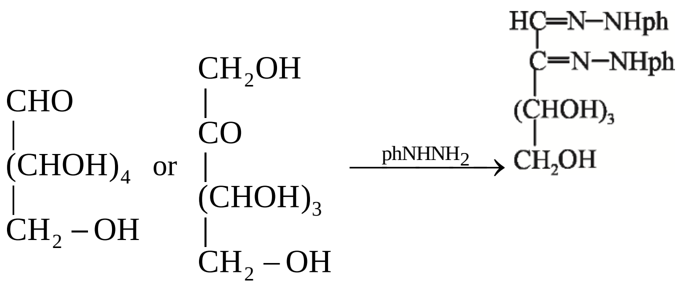
CHEMISTRY

SECTION - 1

1.(D)

2.(B) If both assertion and reason are true but reason is not the correct explanation of the assertion.

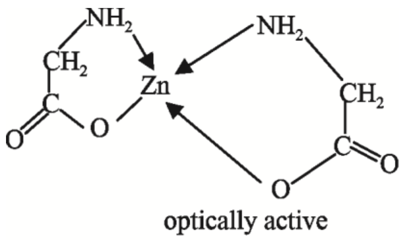
3. (A)



Glucose Fructose

4.(D)

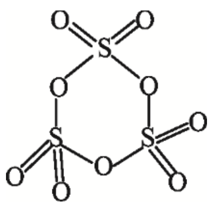
5.(A)



6.(C) $\text{Hydration Energy} \propto \dfrac{1}{r^+}$

7.(C) Gauche form due to H-bonding.

8.(C)

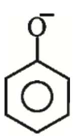


No S-S linkage.

9.(C) As per Fajan's rule.

10.(C) All are aromatic except C

11.(D)



is resonance stablised.

12.(D) $Ph\overset{\oplus}{C}H_2$ and $C=C-\overset{\oplus}{C}H_2$ are resonance stabilized and as phenyl-has more no. of resonance structure.

13.(D) In H_2O (Polar solvent) dibromo product and in CS_2 (non polar) solvent monobromo product is obtained.

14.(B) rate = K[Reactant]^n

15.(D) $CH_3-CH_2-C\equiv CH \xrightarrow[NH_4OH]{Cu_2Cl_2} \underset{(\text{Red ppt})}{CH_3-CH_2-C\equiv C^-Cu^+}$

16.(A) Extent of polarization is higher in group II elements as charge is higher.

17.(C) For exothermic reaction, equilibrium proceeds to endothermic side on increasing temperature.

18.(B)

19.(C) $H_2S_2O_8 \xrightarrow{\text{hydrolysis}} H_2O_2 + H_2SO_4$

20. (B) CN^- is SFL due to which Fe^{3+} configuration will be t_{2g}^5 , hence only 1 unpaired electron . In F^- since it is WFL it will have 5 unpaired electrons.

SECTION - 2

1.(4) 2-Chiral center – 4 compounds

$$\underset{\substack{|\\ Cl}}{H_2C}-H\underset{\substack{|\\ C}}{\dot{C}}-CH_2-CH_3$$

$$H_3C-CH-\overset{\substack{Cl\\ |}}{\underset{\substack{|\\ H}}{C}}-CH_3$$

2.(6) $B_2 \to 0,\ N_2 \to 4, C_2 \to 2$

$0+2+4=6$

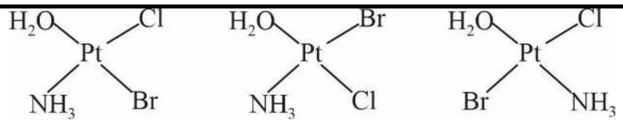
3.(5) $\frac{w}{V}\% = 2.8$

Molarity $= \frac{\frac{2.8}{56}}{1000} \times 1000 = 0.5M$

$10M = 5$

4. (5) $H_2O, NH_3, IF_3, PH_3, D_2O$ have distorted geometry.

5. (2) $a=2, b=4$



$c = 3$

6.(56) $N \xrightarrow{\text{conc.} H_2SO_4} (NH_4)_2 SO_4$

$$\% \text{nitrogen} = \frac{1.4 \times N \times V}{w} = \frac{1.4 \times 1 \times 2 \times 25}{1.25} = 56\%$$

7.(9) $A(g) \quad + \quad 2B(s) \rightleftharpoons 2C(g)$

$t = 0 \quad 2 \quad 4 \quad 0$

$t = t_{eq} \quad 2-1 \quad 2 \quad 2$

$= 1$

$$K_p = \frac{(p_c)^2}{(p_A)} = \frac{(X_C \times P_t)^2}{(X_A \times P_t)} = \frac{\left(\frac{2}{4} \times 9\right)^2}{\left(\frac{1}{4} \times 9\right)} = 9 \text{ atm}$$

8.(278) $1 \times \frac{3}{2} R(T_2 - 300) = -P_{ext}\left(\frac{nRT_2}{P_2} - \frac{nRT_1}{P_1}\right)$

$$1.5(T_2 - 300) = -1\left(\frac{T_2}{3} - \frac{300}{5}\right)$$

$$\frac{3T_2}{2} - 450 = -\frac{T_2}{3} + 60$$

$$\frac{3T_2}{2} + \frac{T_2}{3} = 510$$

$$\frac{11T_2}{6} = 510$$

$$T_2 = 278$$

9.(2) $\lambda_m = \frac{K}{M} = \frac{1}{R} \times \frac{\ell}{A} \times \frac{\text{Volume}}{\text{Mole}}$

$$\frac{1}{100} \times \frac{20}{A} \times \frac{20 \times A}{2}$$

$$= 2\Omega^{-1} cm^2 mol^{-1}$$

10. (40) $\Delta T_f = iK_f m$

$$m = \frac{\frac{0.15}{60}}{\frac{20}{1000}} = 0.125$$

$$\Delta T_f = i\,k_f\,m$$

$$0.5 = i \times 5 \times 0.125$$

$$i = 0.8$$

$$2CH_3COOH \rightleftharpoons (CH_3COOH)_2$$

$$i = 1 + \left(\frac{1}{n} - 1\right)\alpha$$

$$0.8 = 1 + \left(\frac{1}{2} - 1\right)\alpha$$

$$0.2 = \frac{\alpha}{2}$$

$$\alpha = 0.4$$

MATHEMATICS

SECTION-1

1.(B) Distance $CP = CQ = OC = 5\,units$

Slope of OC $= \frac{3}{4}$

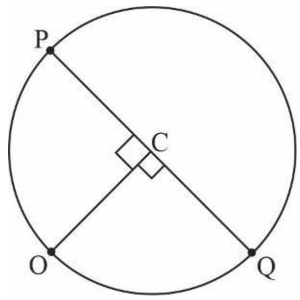
Slope of PQ $= \frac{-4}{3} \Rightarrow \tan\theta = -\frac{4}{3}$

Co-ordinates of point P and Q are $(4 + 5\cos\theta, 3 + 5\sin\theta)$

and $(4 - 5\cos\theta, 3 - 5\sin\theta)$

$= (4 - 3, 3 + 4)$ and $(4 + 3, 3 - 4)$

$(1, 7)$ and $(7, -1)$



2.(A) $\left|\bar{a} + \bar{b} + \bar{c}\right|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2\bar{a}.\bar{b} + 2\bar{b}.\bar{c} + 2\bar{c}.\bar{a}$

$12 = 3|a|^2 + 6|a|^2 \cos\frac{\pi}{3} = 6|a|^2$

$|a| = \sqrt{2}$

Now $\left|2\vec{a} - \vec{b} + \vec{c}\right|^2 = 4|a|^2 + |b|^2 + |c|^2 - 2.\bar{b}.\bar{c} - 4\bar{a}.\bar{b} + 4\bar{a}.\bar{c} = 6|a|^2 - 2|a|^2 \cos\frac{\pi}{3} = 10$

3.(D) ${}^{69}C_{3r-1} + {}^{69}C_{3r} = {}^{69}C_{r^2-1} + {}^{69}C_{r^2}$

$^{70}C_{3r} = {}^{70}C_{r^2}$ .... $[{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$r^2 = 3r$ or $r^2 + 3r = 70$

$r = 0, 3, 7, -10$

Valid values of r = 3 or 7.

4.(C) For point of intersection $x^2 - 2x - 7 = 3x + 7$

$x^2 - 5x - 14 = 0 \Rightarrow x = -2, x = 7$

Area $= \int_{-2}^{7} \left(3x + 7 - \left(x^2 - 2x - 7\right)\right) dx = \frac{243}{2}$

5.(A) $\begin{vmatrix} 1-\lambda & -1 \\ 4 & 2-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 3\lambda + 6 = 0$

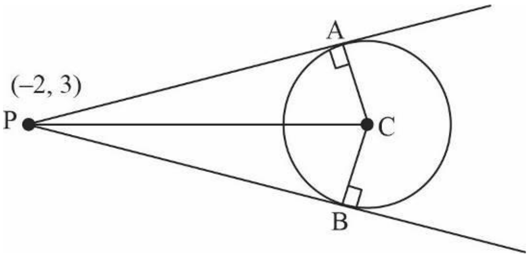
$A^2 - 3A + 6I = 0$

$A^{-1} = \frac{1}{6}(3I - A)$

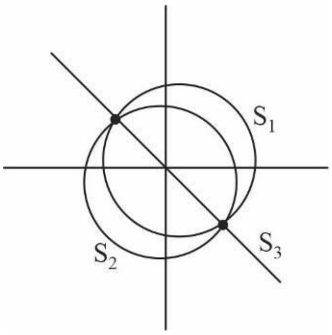
6.(B) $C = (4, -5)$ centre of given circle

Distance PC = diameter of circumcircle of ΔPAB = 10

Area $= \pi r^2 = 25\pi$



7.(C)



8.(C) Let $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda$ .....(i)

Then $x = 3\lambda - 1,\ y = 5\lambda - 3, z = 7\lambda - 5$

General point on this line is $(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$

Again let $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu$ .....(ii)

Then $x = \mu + 2, y = 3\mu + 4, z = 5\mu + 6$

A general point on this line is $(\mu + 2, 3\mu + 4, 5\mu + 6)$

For intersection, they have a common point, for some values of λ and μ, we must have $bcx + cay - abz = 0$,

$(5\lambda - 3) = (3\mu + 4), \quad (7\lambda - 5) = (5\mu + 6)$

From first two we have, $\mu = 3\lambda - 3$ ……(iii) and $3\mu = 5\lambda - 7$ .....(iv)

From (iii), put the values of μ in (iv), we have $3(3\lambda - 3) = 5\lambda - 7$

$\Rightarrow 9\lambda - 9 = 5\lambda - 7$ or $4\lambda = 2$ or $\lambda = \frac{1}{2}$

Put $\lambda = \frac{1}{2}$ in (iii), we get $\mu = -\frac{3}{2}$ (Putting $\lambda = \frac{1}{2}$)

The required point of intersection is

$$\left[\frac{3}{2}-1\right],\left[\frac{5}{2}-3\right],\left[\frac{7}{2}-5\right]=\left[\frac{1}{2},-\frac{1}{2},-\frac{3}{2}\right].$$

9.(B) $D \ge 0 \Rightarrow 16 - 4(m-3) \ge 0 \Rightarrow m \le 7$

$\alpha + \beta < 6 \Rightarrow (-4) < 6 \Rightarrow m \in R$

And $f(3) > 0 \Rightarrow 9 + 12 + m - 3 > 0 \Rightarrow m > -18$

$\Rightarrow m \in (-18, 7]$

$x^2 + 4x + (m-3) = 0$

10. (C) $\lim\limits_{x \to 1}\left(3 - x + a[x-2] + b[x+2]\right)$

$\lim\limits_{x \to 1}\left((a+b)[x] + 3 - x + 2b - 2a\right)$ exists

$\Rightarrow a + b = 0,$ as $[x]$ is discontinuous at integer.

11.(A) $\left(\frac{2a-3}{2}\right)^2 + \left(\frac{a-7}{2}\right)^2 + 3 \le 49$

$5a^2 - 26a - 126 \le 0$

$a \in \left[\frac{26-\sqrt{3196}}{10}, \frac{26+\sqrt{3196}}{10}\right]$

$a \in \{-3, -2, -1, ..., 8\}$

12 integers

12.(A) $\bar{x} = \frac{1}{n}\sum(2x_i + 5) = 2\alpha + 5$

$$\sigma^2 = \frac{1}{n}\sum\left(2x_i + 5 - (2\alpha + 5)\right)^2 = \frac{4}{n}\sum(x_i - \alpha)^2 = 4\beta$$

13.(B) $\frac{xdy - ydx}{x^2} = \left(xe^x + e^x - 1\right)dx$

$\frac{y}{x} = xe^x - x + c$

$x=1 \Rightarrow y=e-1 \Rightarrow c=0$

$y=x^2\left(e^x-1\right)$

14.(C) For real solutions

$(2a-7)^2+(a+3)^2 \geq (3a-1)^2$

$4a^2+16a-57 \leq 0$

$a \in \left[\frac{-4-\sqrt{73}}{2}, \frac{-4+\sqrt{73}}{2}\right]$

$a \in \{-6,-5,.........2\}$

15.(B) No of ways of getting sum 10 is

$=$ coefficient of x^{10} is $\left(x+x^2+.....x^6\right)^3=27$

Probability $=\frac{27}{216}=\frac{1}{8}$

16.(B) Point of intersection lies on $\left(\frac{x^2}{a^2}+y^2-1\right)+\left(x^2+\frac{y^2}{a^2}-1\right)=0$ -----[Concept of family of circle]

$x^2+y^2=\frac{2a^2}{1+a^2}$

17.(C) $s_1=\frac{1}{1+\alpha}, s_2=\frac{1}{1+\beta}$

Let $s=1-\alpha\beta+\alpha^2\beta^2.......\Rightarrow s=\frac{1}{1+\alpha\beta}$

$\alpha=\frac{1}{s_1}-1,\ \beta=\frac{1}{s_2}-1;\ \therefore s=\frac{1}{1+\left(\frac{1}{s_1}-1\right)\left(\frac{1}{s_2}-1\right)}$.

$s=\frac{s_1s_2}{2s_1s_2+1-s_1-s_2}$.

18.(A) $\lim_{x\to 2}\frac{x^2 f(2)-4f(x)}{x-2}=\lim_{x\to 2}\frac{2xf(2)-4f'(x)}{1}=4f(2)-4f'(2)=4\times 3-4(-1)=16$

19. (B) For continuity at $x=3$ $\quad 9a-3b+2=9b-3$

For differentiability $\quad 6a-b=6b$

$a=\frac{35}{9}, b=\frac{10}{3}$

20.(A) $\int x^2e^{2x}dx=x^2.\frac{e^{2x}}{2}-2x.\frac{e^{2x}}{4}+2\frac{e^{2x}}{8}+c$

$=\frac{1}{4}\left(2x^2-2x+1\right)e^{2x}+c$

$$f(x)=\frac{1}{4}\left(2x^2-2x+1\right)$$

Min value of $f(x)$ is $\frac{1}{8}$

SECTION-2

1.(40) Diagonals are $\vec{p}=\vec{a}+\vec{b}=4\vec{\alpha}+2\vec{\beta}$ and $\vec{q}=\vec{a}-\vec{b}=2\vec{\alpha}-4\vec{\beta}$

$$|\vec{p}|^2=16+4+16\cos\frac{\pi}{3}=28$$

$$|\vec{q}|^2=4+16-16\cos\frac{\pi}{3}=12$$

2.(1) Solving the determinant $=6i\left(3i^2+3i\right)+3i\left(4i+20i\right)+\left(12-60i\right)$

$$x+iy=-78(1+i)$$

3.(8) $\int_0^x f(t)dt+x\int_0^x f(t)dt-\int_0^x tf(t)dt=e^{-x}-1+x$

Applying Leibniz theorem

$$f(x)+1.\int_0^x f(t)dt+xf(x)-xf(x)=-e^{-x}+1$$

$$f(x)+\int_0^x f(t)dt=-e^{-x}+1$$

$$f'(x)+f(x)=e^{-x}$$

$$f(x)e^x+e^x f'(x)=1$$

Integrating both sides $f(x)e^x=x+c$

$$f(0)=0\Rightarrow c=0\Rightarrow f(x)e^x=x$$

4.(4) $\vec{a}.\vec{b}+\vec{a}.\vec{c}=0,\vec{b}.\vec{c}+\vec{b}.\vec{a}=0,\vec{c}.\vec{a}+\vec{c}.\vec{b}=0$

$$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$$

Now $\left|\vec{a}+\vec{b}+\vec{c}\right|^2=|\vec{a}|^2+\left|\vec{b}\right|^2+|\vec{c}|^2+0=2^2+3^2+6^2=7^2$

5. (2) $7\{x\}=12-4[x]=$ integer

$$\{x\}=0,\frac{1}{7},\frac{2}{7},\frac{3}{7},\frac{4}{7},\frac{5}{7},\frac{6}{7}$$

$$[x]=\frac{12}{4},\frac{11}{4},\frac{10}{4},\frac{9}{4},\frac{8}{4},\frac{7}{4},\frac{6}{4}$$

Two possibilities only

6.(4) $S=\frac{1}{1.3.5}+\frac{1}{3.5.7}+\frac{1}{5.7.9}+.......\infty$

$$S=\frac{1}{4}\left[\left(\frac{1}{1\times3}-\frac{1}{3\times5}\right)+\left(\frac{1}{3\times5}-\frac{1}{5\times7}\right)+....\right]$$

$$S=\frac{1}{12}$$

7.(9) $\begin{vmatrix} 1 & 1 & -1 \\ 2 & a & 1 \\ 3 & -2 & 7 \end{vmatrix} = 0$

$a = \frac{1}{2}$

8.(5) Last digit in $3^m = 3, 9, 7, 1$

And last digit in $7^n = 7, 9, 3, 1$

Hence $3^{4k} + 7^{4k+2}\, or\, 3^{4k+2} + 7^{4k}$

Or $3^{4k+1} + 7^{4k+3}\, or\, 3^{4k+3} + 7^{4k+1}$ type

Numbers will be divisible by 10

$k = 5\times5 + 5\times5 + 5\times5 + 5\times5 = 100$

9.(7) $\frac{dy}{dx} + y(-3\cot x) = \sin 2x$

Integrating factor $= e^{\int -3\cot x dx} = e^{-3\log \sin x} = \frac{1}{\sin^3 x}$

$y.\frac{1}{\sin^3 x} = \int \frac{\sin 2x}{\sin^3 x} dx = \int 2\cot x \cos ec x dx = -2\cos ec x + c$

$y = f(x) = c.\sin^3 x - 2\sin^2 x$

$f\left(\frac{\pi}{2}\right) = 3 = c - 2 \Rightarrow c = 5$

$f\left(-\frac{\pi}{2}\right) = 5(-1) - 2(1) = -7$

10.(3) as $x \to 0;\ \frac{\sin x}{x} \to 1$ but less than 1

Other 3 approaches to 1 but more than 1